**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. **0.2659**
4. 0.5
5. 0.6987

Ans: Mean (m) = 45 minutes

Std (s ) = 8 minutes

X= 60-10 =50 minutes

Z = (X-μ)/s

= (50-45)/8

= 0.625

from scipy import stats

[ 1-stats.norm.cdf(abs(0.625)) ]

OR

[ 1-stats.norm.cdf(50,45,8) ]

= [0.26598552904870054]

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Ans: **False**

[ 1-stats.norm.cdf(44,38,6) ] = [0.15865525393145707]

stats.norm.cdf(44,38,6)-stats.norm.cdf(38,38,6)

= 0.3413447460685429

OR

Around 70% of the data falls within one standard deviation of the mean (µ+s= 38+6=44)

If more employees are older than 44, this will shift the m towards 44 with considering sd. Which is not possible as m is given 38 with Sd 6.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans: **True**

Employees under 30 yrs of age P(X≤30) :

stats.norm.cdf (30,38,6)

[0.09121121972586788]

OR

Z=(X-µ)/ s

Employees under 30 yrs of age P(X≤30) :

P(X≤30) =p (Z ≤ (30-38) /6)

= p (Z ≤ -1.33)

= 0.0918( by z table)

Expected count=0.0912\*400= 36.72

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans: if X ∼ N (µ1, σ1^2) And Y ∼ N (µ2, σ2^2) are two independent random variables

then

X + Y ∼ N (µ1 + µ2, σ1^2 + σ2^2) And X − Y ∼ N (µ1 − µ2, σ1^2 + σ2^2)

Similarly, if Z = aX + bY i.e Z is linear combination of X and Y,

then

Z ∼ N (aµ1 + bµ2, a^2σ1^2 + b^2σ2^2

Therefore

2X1~ N (2 u,4 σ^2)

X1+X2 ~ N (µ + µ, σ^2 + σ^2) ~ N (2 u, 2σ^2)

2X1-(X1+X2) = N (4µ,6 σ^2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. **48.5, 151.5**
6. 90.1, 109.9

Ans: stats.norm.interval(0.99,100,20)

(48.48341392902199, 151.516586070978)

OR

The Probability of getting value between a and b is 0.99

Probability outside the a and b area is 0.01 (i.e. 1- 0.99).

0.01/2 =0.005

The Probability towards left from a = -0.005

The Probability towards Right from b = +0.005

For Probability 0.005 the Z Value is -2.57 (by Z Table)

Z =(X- μ) / σ

X = Z \* σ + μ

Z(-0.005)\*20+100 = 151.4

Z(+0.005)\*20+100 = 48.6

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans: import numpy as np

from scipy import stats

from scipy.stats import norm

Mean = 5+7

print('Mean Profit is Rs', Mean\*45,'Million')

Mean Profit is Rs 540 Million

SD = np.sqrt((9)+(16))

print('Standard Deviation is Rs', SD\*45, 'Million')

Standard Deviation is Rs 225.0 Million

print('Range is Rs',(stats.norm.interval(0.95,540,225)),'in Millions')

Range is Rs (99.00810347848784, 980.9918965215122) in Millions

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans: To compute 5 th percentile ,we use the formula X = μ + Zσ; wherein

from z table, 5 th percentile = -1.645

X= 540+(-1.645) \* (225)

Therefore, 5th percentile of profit (in Million) is 170.00

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: Using stats.norm.cdf(0,5,3) Probability of Division 1 making a loss P(X<0)

is 0.04780 and stats.norm.cdf (0,7,4) probability of Division 2 making a loss

P(X<0) is 0.04005.

Ans.is Division 1 has Larger Probability of Making Loss in a given year